

CS5000 Fall 2008
Assignment 11
Due: December 1, 2008 by 11:59 pm
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Problem 1 (2 points)

Let $\neg HALT(x, y) = \begin{cases} 1 & \text{if } \Phi(x, y) \uparrow \\ 0 & \text{if } \Phi(x, y) \downarrow \end{cases}$. Prove or disprove: $\neg HALT(x, y)$ is computable.

Problem 2 (2 points)

Suppose someone claims that a particular program P computes $HALT(x, x)$. Give an input to P on which P gives an incorrect result.

Problem 3 (8 points)

Goldbach's conjecture is an old unsolved problem in number theory. An interesting Wiki article about it is at http://en.wikipedia.org/wiki/Goldbach's_conjecture. Note that the article has a few typos. The conjecture states that every even number greater than or equal to 4 is the sum of two primes. For example, $4 = 2 + 2$, $6 = 3 + 3$, $8 = 3 + 5$, $10 = 5 + 5$, $12 = 5 + 7$, etc. Consider the following function:

$$f(x) = \begin{cases} x & \text{if Goldbach's conjecture is true;} \\ x & \text{if } x < z; \\ 0 & \text{if } x \geq z, \end{cases}$$

where z is the smallest number for which Goldbach's conjecture is false. Prove or disprove: $f(x)$ is primitive recursive.