CS 5000: Lecture 12

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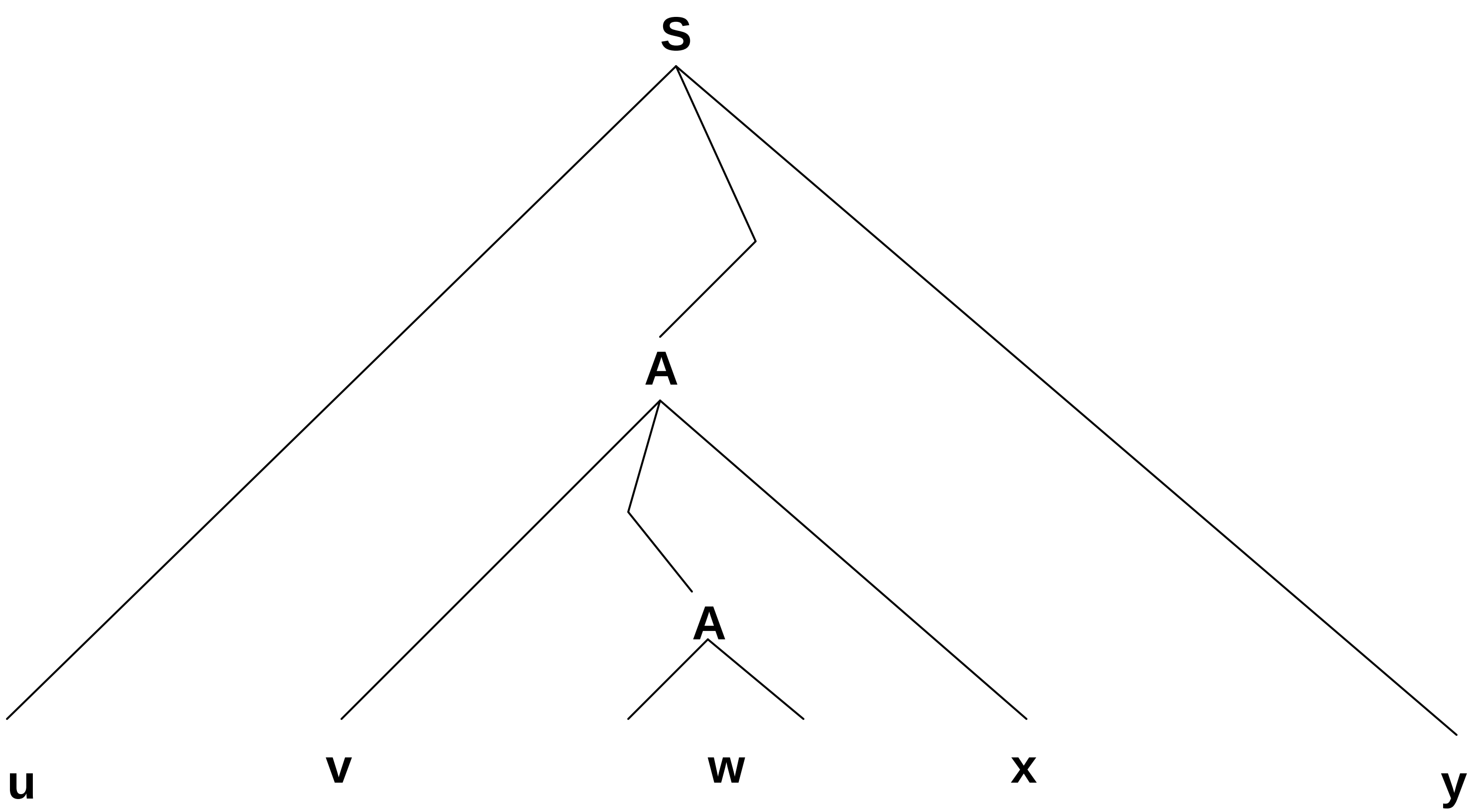
Outline

- Pumping Lemma for Context-Free Languages

Pumping Parse Tree



Pumping Parse Tree: Definition

- A pumping parse tree for a CFG G = (V, Σ, S, P) is a parse tree that satisfies two properties:
  1. There exists some variable A such that A's node in the tree has itself as its own descendant.
  2. The yield generated by the ancestor A is greater than the yield generated by the descendant A.

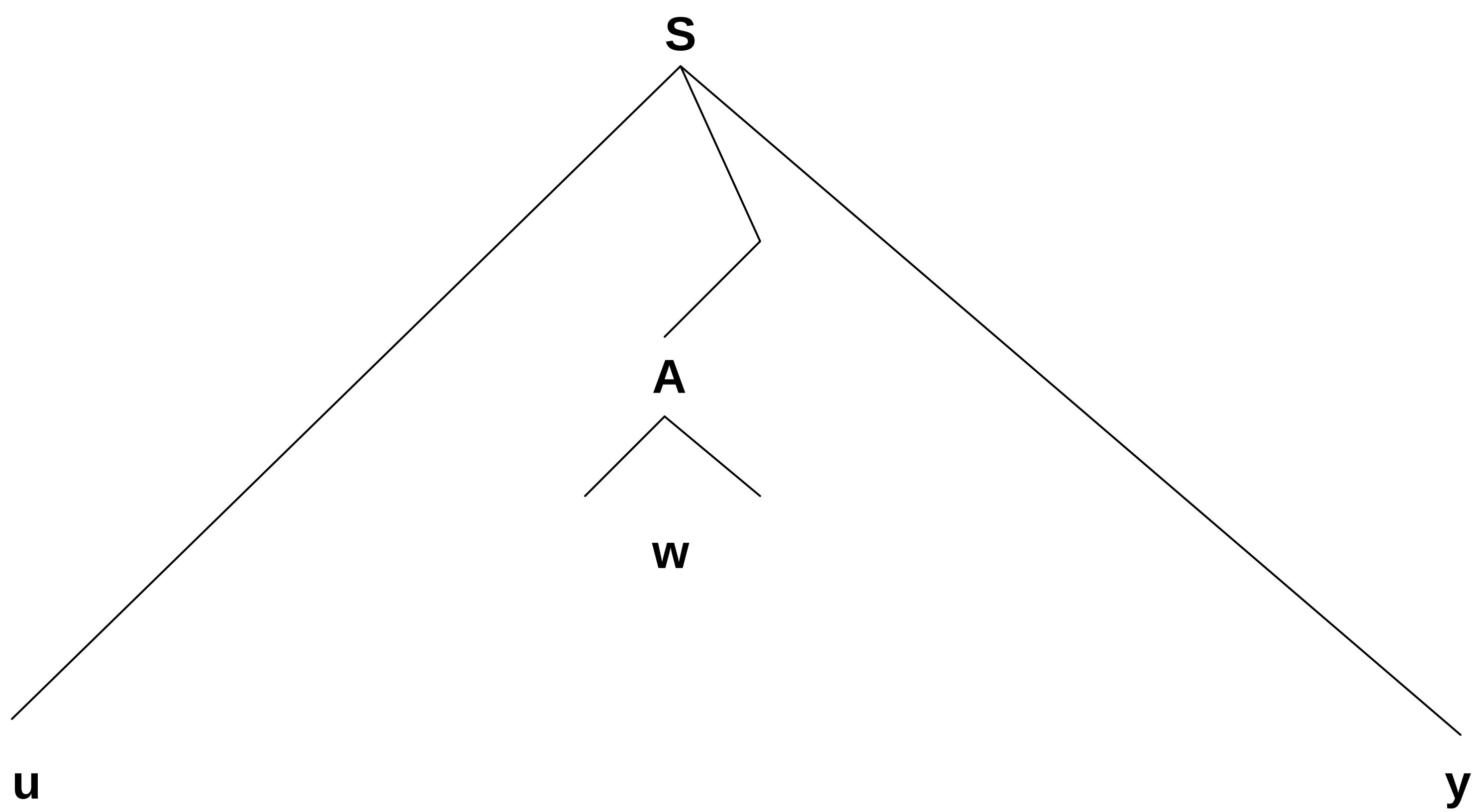
Pumping Parse Trees

Theorem: If a context-free grammar ***G*** generates a pumping parse tree with a yield as shown on the previous slide, then uv^iwx^iy is in ***L(G)*** for all $i >= 0$.

Proof Sketch

- Consider the string uv^0wx^0y.
- The tree that generates this string can be obtained from the parse tree for *uvwxy* by replacing the ancestor node with the descendant node.
- The pumped string is still in the language.

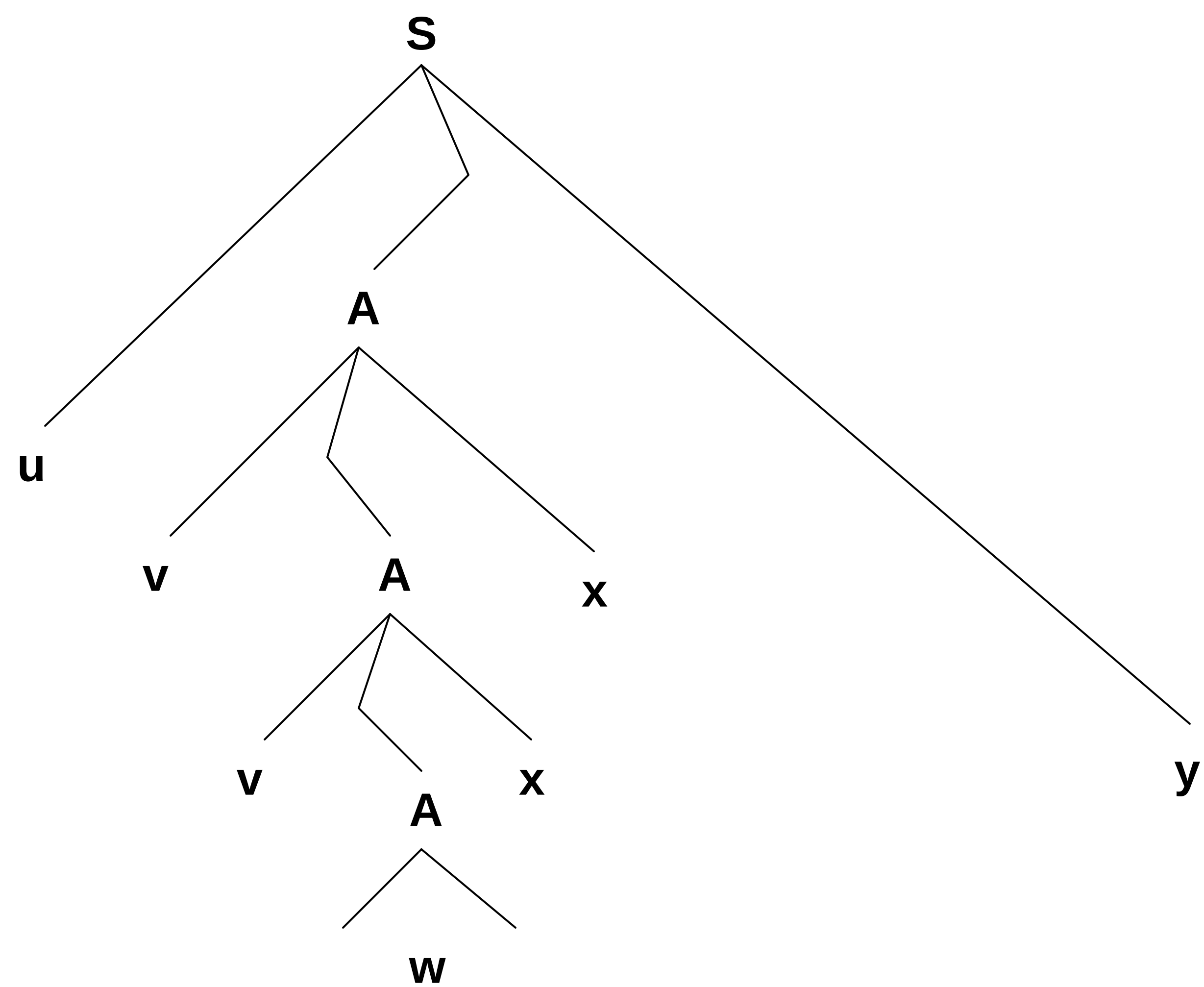
Parse Tree for *uvw*



Proof Sketch

- Consider the string uv^2wx^2y.
- The tree that generates this string can be obtained from the parse tree for *uvwxy* by replacing the descendant node with the ancestor node.
- The pumped string is still in the language.

Pumping Parse Tree for *uvvwxxy*



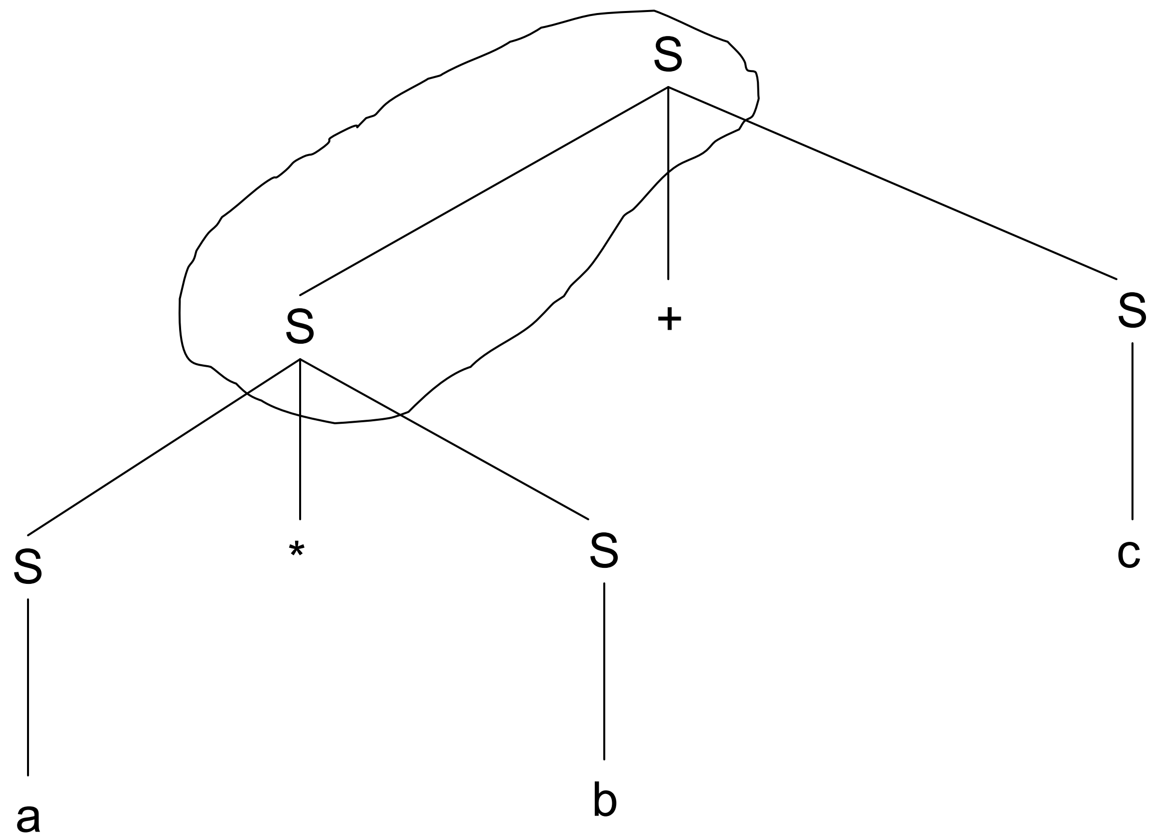
Proof Sketch

- In general, consider the string uv^iwx^iy.
- The tree that generates this string can be obtained from the parse tree for *uvwxy* by replacing the descendant node with the ancestor node *i-1* times.
- The pumped string is still in the language.

Example

- Consider the following grammar.
- S → S
- S → S + S
- S → S * S
- S → a
- S → b
- S → c

Example



Choosing the Pumping Constant

Theorem: For every CFG G = (V, Σ, S, P) there exists some k greater than the length of any string generated by a parse tree of height |V| + 1.

Proof Sketch

For any CFG G, there are only finitely many parse trees of height |V| + 1. Each of those parse trees will be a pumping parse tree, by the pigeonhole principle. Take k to be the length of the longest string generated by these parse trees plus 1.

CFL Pumping Lemma

- Theorem: For all CF languages L there exists some natural number k such that for all z in L such that $|z| >= k$, there exists some $uvwxy$ such that
  1. $z = uvwxy$;
  2. v and x are not both empty;
  3. $|vwx| <= k$;
  4. for all i, uv^iwx^iy is in L.

Proof Sketch

- Consider a parse tree of height |V| + 1.
- That tree will be a pumping parse tree.
- Once we know that it is a pumping parse tree, we can use the same argument as in the theorem about the pumping parse trees.