CS 5000: Lecture 6

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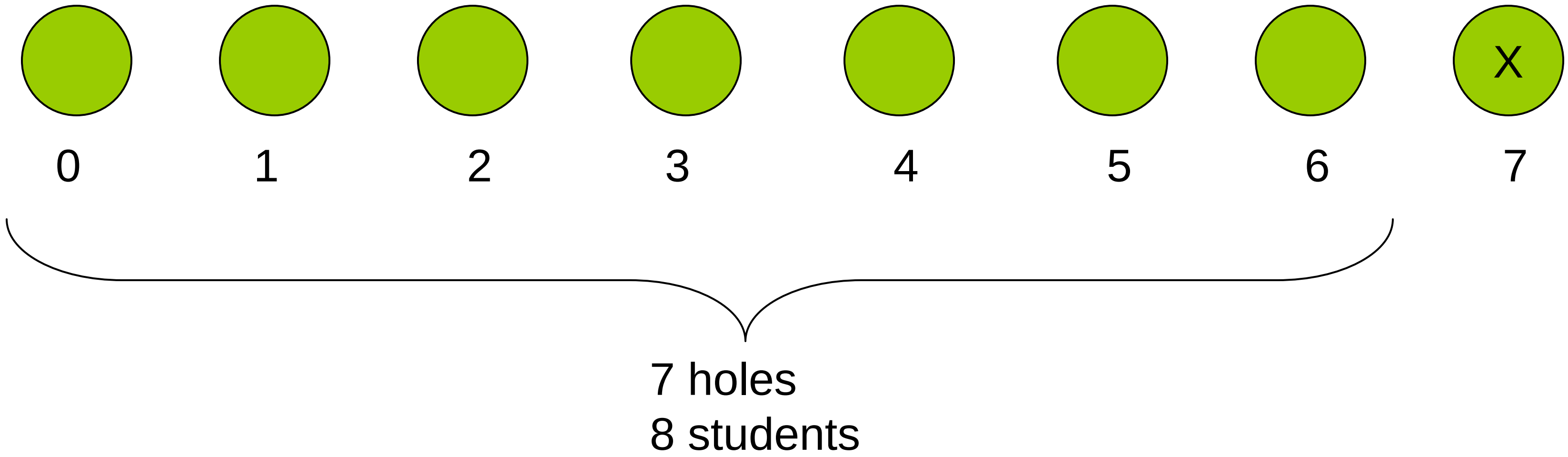
Problem

9 students take a test. Student X makes 7 errors. Each of the other students makes fewer errors. Did at least two students make the same number of errors?

Solution



Pigeon Hole Principle

Formulation 1: If ***(n+1)*** pigeons fly through ***n*** holes, there is at least ***one*** hole through which at least ***two*** pigeons fly.

Formulation 2: If ***(n+1)*** objects are distributed among ***n*** sets, at least ***one*** set must contain at least ***two*** objects.

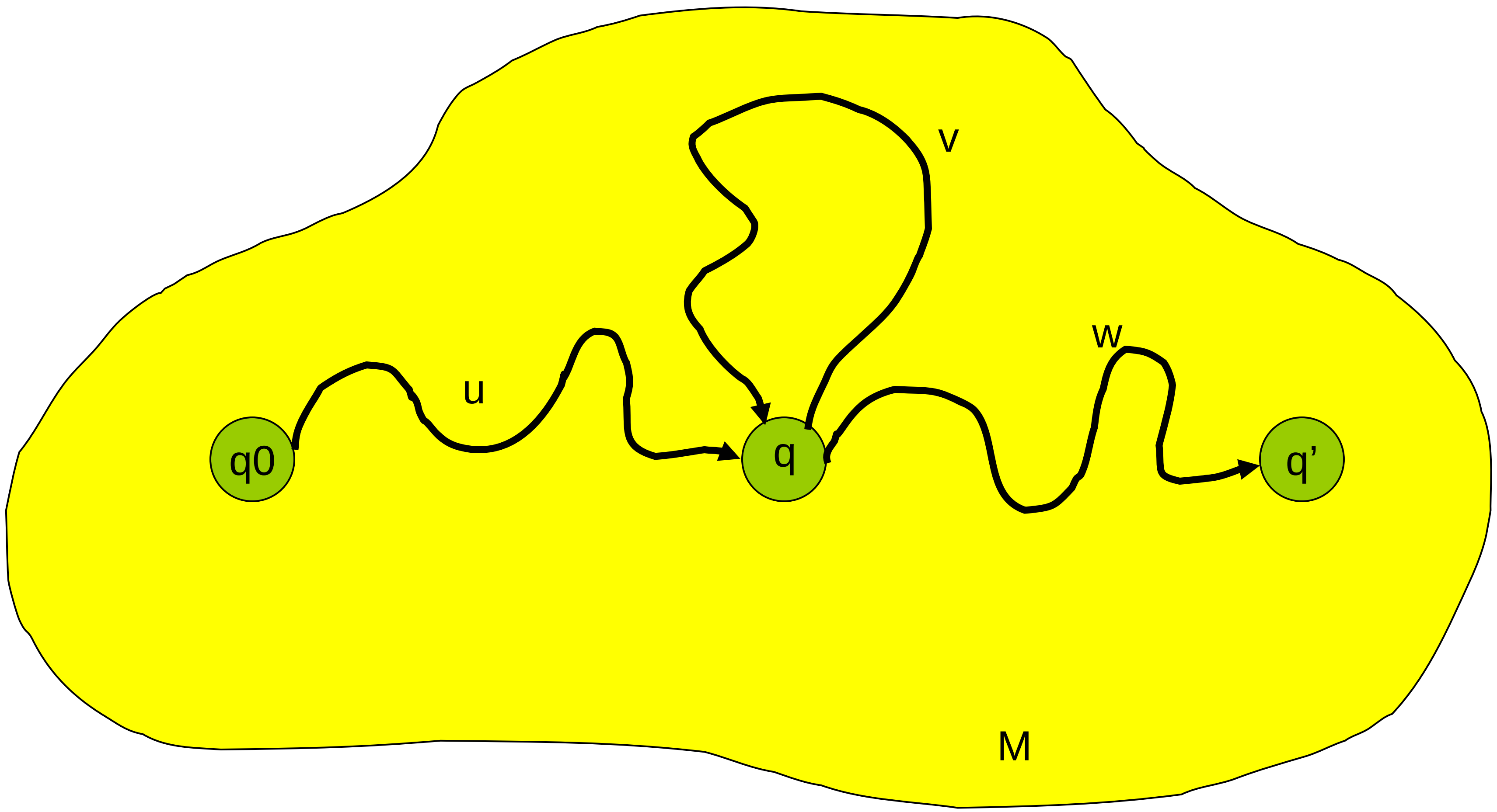
Theorem 6.1 (Ch. 9): The Pumping Lemma

Let $L = L(M)$, where M is a dfa with n states. Let $x \in L$ and $|\mathrm{x}| \geq n$. Then we can write $x = uvw$, where $1 \leq |v| \leq n$ and $uv^i w \in L$, for $i \geq 0$.

Proof

- ***x*** has at least ***n*** symbols.
- ***M*** has exactly ***n*** states.
- ***n+1*** states are required to process ***n*** symbols.
- Since ***M*** has exactly ***n*** states, as it processes ***x***, it must enter the same state at least twice.

Proof



Proof

$$\delta^*(q_0, u) = q.$$

$$\delta^*(q, v) = q.$$

$$\delta^*(q, w) = q' \in F.$$

Proof

1. $v \neq 0.$

2. $\delta^*(q_0, uvw) = \delta^*(q_0, uw) \in F.$

3. $\delta^*(q_0, uvw) = \delta^*(q_0, uvvw) = \delta^*(q_0, uv^i w) \in F.$

4. $v \neq 0$ can be pumped $0, 1, 2, 3...$ times and the string $uv^i w$ will still be in L.

Example 1

Claim: $\{a^k b^k \mid k \in N\}$ is not regular.

Proof: Let n be the constant of the Pumping Lemma.

Let $z \in L, |z| \geq n$. Then $z = uvw, 1 \leq |v| \leq n$. Then

$z = a^m b^m, m > 0$. There are 3 choices for $v: v = a^+;$

$v = a^+ b^+; v = b^+$. If $v = a^+, uv^0 w = a^k b^m, k < m$. Thus,

$uv^0 w \notin L$. If $v = a^+ b^+, uv^2 w$ is a string where a's follow

b's. Thus, $uv^2 w \notin L$. If $v = b^+, uv^0 w = a^m b^k, m > k$. Thus,

$uv^0 w \notin L$. In each of the 3 cases, we achieve a contradiction.

Example 2

Claim: $L = \left\{ a^{i^2} | i \in N \right\}$ is not regular.

Proof : Assume that L is regular. Let n be the constant of the Pumping Lemma. Let $z = a^{n^2}$. Let $z = uvw, 1 \leq |v| \leq n$. Then $n^2 < |uv^2w| \leq n^2 + n < (n+1)^2$. Thus, $uv^2w \notin L$.

Suggested Reading

- Chapter 9, Section 6.