CS 5000: Lecture 4

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Outline

- Nondeterminism
- Power Sets
- Nondeterministic FA (NFA)
- NFAs and their Languages
- Instantaneous Descriptions
- DFA → NFA
- NFA → DFA

Nondeterminism

- Given an input, there can be more than one legal sequence of steps to process the input.
- The input is accepted if at least one legal sequence of moves ends up in an accepting state.

Power Sets

Let S be a set. The power set of S is

$$P(S) = \{R \mid R \subseteq S\}.$$

Nondeterministic Finite Automata

An NFA M is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$,

where

Q is a finite set of states;

Σ is an alphabet, i.e. a finite set of symbols;

$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to P(Q)$;

$q_0 \in Q$ is the start state;

$F \subseteq Q$ is the set of accepting states.

Instantaneous Descriptions (IDs)

An ID for an NFA is a pair (q, x) where $q \in Q$ is the current state and $x \in \Sigma^*$ is the unread part of the input.

For any string $x \in \Sigma^*$ and any $a \in \Sigma$ or $a = \varepsilon$, $(q, ax) \mapsto (r, x)$ if and only if $r \in \delta(q, x)$.

$\rightarrow^*$ is a relation on IDs, i.e. $I \rightarrow^* J$ if and only if there is a sequence of zero or more $\rightarrow$ relations that starts with I and ends with J.

NFAs and Their Languages

$$\delta^*(q,x) = \{r \mid (q,x) \mapsto^* (r,\varepsilon)\};$$

If M is an NFA, then $L(M) = \{x \in \Sigma^* | \delta^*(q_0,x) \cap F \neq \varnothing\}.$

NFA ➔ DFA

Theorem : Let N be a NFA. Then there is a DFA D such that $L(N) = L(D)$ is regular.

Proof : Let $N = (Q_N, \Sigma, \delta_N, q_N, F_N)$. Construct a new DFA $D = (Q_D, \Sigma, \delta_D, q_D, F_D)$ where

1. $Q_D = P(Q_D)$;
2. $\delta_D(R, a) = \bigcup_{r \in R} \delta_N^*(r, a), R \in Q_D, a \in \Sigma$;
3. $\delta_N^*(q_N, \varepsilon) = q_D$;
4. $F_D = \{R \in Q_D \mid R \cap F_N \neq \varnothing\}$.

By construction, $\delta_D^*(q_D, x) = \delta_N^*(q_N, x)$, for all $x \in \Sigma^*$.

DFA ➔ NFA

Theorem (DFA $\rightarrow$ NFA): If L is regular, there is some NFA N such that $L(N) = L$.

Proof : Let L be a regular language. There must be some DFA $D = (Q, \Sigma, \delta_D, q_0, F)$ with $L(D) = L$. We can make D into an NFA by defining $\delta_N(q, a) = \{\delta_D(q, a)\}$, for $q \in Q$ and $a \in \Sigma$.