CS 6100 Homework 4 (15 points)

This assignment can be done in groups of one or two.

1. In the following strategic-form game, what strategies survive iterated elimination of strictly-dominated strategies? What are the pure-strategy Nash equilibria?

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

2. Agents 1 and 2 are bargaining over how to split a dollar. Each agent simultaneously names shares they would like to have (s1 and s2) where $0 \leq s1 \leq 1$ and $0 \leq s2 \leq 1$. If $s1+s2 \leq 1$ then both agents receive the shares they named; if $s1+s2 > 1$, then both agents receive zero. Draw the best response function for both players (like is in the notes, at the last of Wool6.ppt). What are the pure strategy equilibrium of this game? Hint, to make it easier, consider a fixed set of choices, say 0, .2, .4, .6, .8, 1 for each player.

Hint: A best response function says, "If player A does x, what is player B's response (shown on y axis)." And conversely, if player B does y, what is player A's response (shown on x axis). It works out nicely if you can draw both functions on the same set of axis. The point where they cross is equilibrium.

3. At a fishing booth at a carnival, two children randomly get prizes. There are 5 types of prizes of varying values. Assume, a prize of type 5 is the best and a prize of type 1 is the worst. They both get a prize that they don't show to other. All prizes occur with the same frequency, so they don't assume there are more of the bad prizes. They are both asked if they want to exchange the prizes they were given. If both want to exchange, the two children exchange prizes. Otherwise, they keep what they were given. Model this situation as a strategic form game and show that in any Nash equilibrium, the highest prize that either player is willing to exchange is the smallest possible prize.

4. For the matrix game below, apply iterated elimination of dominated strategies. Indicate whether or not you can solve the game by this method. If there is a solution, is it pareto optimal?

	c_1	c_2	c_3	c_4
r_1	−1, −1	−1, −1	5, −3	3, 4
r_2	4, 1	0, 5	2, 0	5, 1
r_3	3, 6	−1, 0	4, 4	4, 0
r_4	0, 0	0, 0	0, 0	0, 0

5. Consider the strategic form game below, shown in bimatrix form (we just split the rewards into two matrices rather than show them as one).

(a) Is there a Nash Equilibrium in pure strategies?

(b) Find the Nash equilibrium with mixed strategies.

$$A = \begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}.$$

6. Consider the bimatrix game below: Find the Nash equilibrium with mixed strategies.
You haven't seen an example quite like this, but its just more of the 2x2 case. Hint, the probabilities are p_1, p_2, and $(1-p_1-p_2)$ and q1,q2 and (1-q1-q2)

$$A = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 1 \\ 3 & 0 & 0 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$