CS 6100 Homework 2

This assignment can be done in groups of one, two or three.

1. In the following strategic-form game, what strategies survive iterated elimination of strictly-dominated strategies? What are the pure-strategy Nash equilibria?

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

2. Agents 1 and 2 are bargaining over how to split a dollar. Each agent simultaneously names shares they would like to have (s1 and s2) where $0 \leq s1 \leq 1$ and $0 \leq s2 \leq 1$. If $s1+s2 \leq 1$ then both agents receive the shares they named; if $s1+s2 > 1$, then both agents receive zero. Draw the best response function for both players (like is in the notes, at the last of Wool6.ppt). What are the pure strategy equilibrium of this game? Hint, to make it easier, consider a fixed set of choices, say 0, .2, .4, .6, .8, 1 for each player.

Hint: A best response function says, "If player A does x, what is player B's response (shown on y axis)." And conversely, if player B does y, what is player A's response (shown on x axis). It works out nicely if you can draw both functions on the same set of axis. The point where they cross is equilibrium.

3. At a fishing booth at a carnival, two children randomly get prizes. There are 5 types of prizes of varying values. Assume, a prize of type 5 is the best and a prize of type 1 is the worst. They both get a prize that they don't show to other. All prizes occur with the same frequency, so they don't assume there are more of the bad prizes. They are both asked if they want to exchange the prizes they were given. If both want to exchange, the two children exchange prizes. Otherwise, they keep what they were given. Model this situation as a strategic form game and show that in any Nash equilibrium, the highest prize that either player is willing to exchange is the smallest possible prize.

4. For the matrix game below, apply iterated elimination of dominated strategies. Indicate whether or not you can solve the game by this method. If there is a solution, is it pareto optimal?

	c_1	c_2	c_3	c_4
r_1	−1, −1	−1, −1	5, −3	3, 4
r_2	4, 1	0, 5	2, 0	5, 1
r_3	3, 6	−1, 0	4, 4	4, 0
r_4	0, 0	0, 0	0, 0	0, 0

5. Consider the strategic form game below, shown in bi-matrix form (we just split the rewards into two matrices rather than show them as one).

(a) Is there a Nash Equilibrium in pure strategies?

(b) Find the Nash equilibrium with mixed strategies.

$$A = \begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}.$$

6. Consider the bi-matrix version of the game below: Find the Nash equilibrium with mixed strategies. You haven't seen an example quite like this, but its just more of the 2x2 case. Hint, the probabilities are p_1, p_2, and $(1-p_1-p_2)$ and q1,q2 and (1-q1-q2)

$$A = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 1 \\ 3 & 0 & 0 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 0 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

7. Analyze the diner's dilemma for stability. The **Diner's dilemma** is an n-player Prisoner's dilemma. The situation imagined is several individuals go out to eat, prior to ordering they agree to split the check equally between all of them. Each individual must now choose whether to order the expensive or inexpensive dish. It is supposed that the expensive dish is better than the cheaper, but not better enough to warrant paying the difference if you were eating alone. Each individual reasons that the added expense she adds to her bill by ordering the more expensive item is very small, and thus the improved gustatory experience is worth the money. However, every individual reasons this way and they all end up paying for the cost of the more expensive meal, which, by hypothesis, is worse for everyone than ordering and paying for the cheaper meal.

8. For the Battle of the Sexes interaction shown below, if the row player selects Opera with a 2/3 probability, what is the column player's best response?

a) Opera
b) Football
c) All responses are equally good
d) Opera with a probability of 1/3

Battle of the Sexes		
	Opera	Football
Opera	3, 2	1, 1
Football	0, 0	2, 3

9. In the following strategic-form game, what strategy can be eliminated via iterated elimination of strictly-dominated strategies?

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

a) Row T
b) Row M
c) Row B

d) Column L
e) None of the above

10. Two companies (A and B) sell mineral water. Each company has a fixed cost of $5000 per period, regardless of how many bottles they produce. The two companies are competing for the same market and each firm must choose a high price ($2 per bottle) or a low price ($1 per bottle). Here are the rules of the game:
1) At a price of $2 per bottle, 5000 bottles can be sold for a total revenue of $10000.
2) At a price of $1 per bottle, 10000 bottles can be sold for a total revenue of $10000.
3) If both companies charge the same price, they split the sales evenly between them.
4) If one company charges a higher price, the company with the lower price sells the whole amount and the company with the higher price sells nothing.
5) Payoffs are profits -- revenue minus the $5000 fixed cost.

Fill in the table below with utilities and indicate what will likely happen:

	Cheap (B)	Expensive (B)
Cheap (A)		
Expensive (A)		